LOGICAL DEDUCTION IN AI

INFERENCING BY RESOLUTION REFUTATION

Partha P Chakrabarti
Indian Institute of Technology Kharagpur

Solve the “Alpine Club Problem” using FOL

Tony, Mike, and John belong to the Alpine Club. Every member of the Alpine Club is either a skier, a mountain climber, or both. No mountain climber likes rain, and all skiers like snow. Mike dislikes whatever types of weather Tony likes and likes whatever types of weather Tony dislikes. Tony likes rain and snow. Is there a member of the Alpine Club who is a mountain climber but not a skier? If so, who?

Alpine Club Problem

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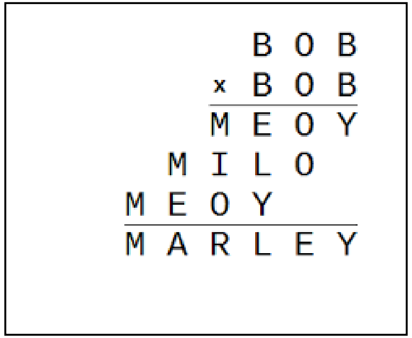
SEARCH IN AI

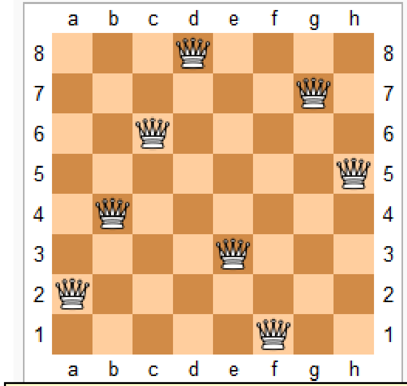
CONSTRAINT SATISFACTION PROBLEMS

Partha P Chakrabarti
Indian Institute of Technology Kharagpur

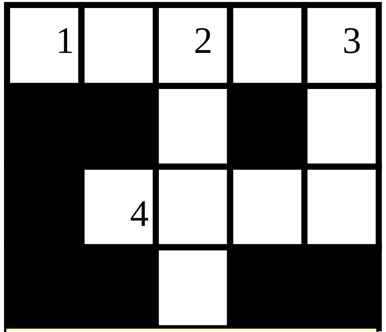
Constraint Satisfaction Problems (CSPs)



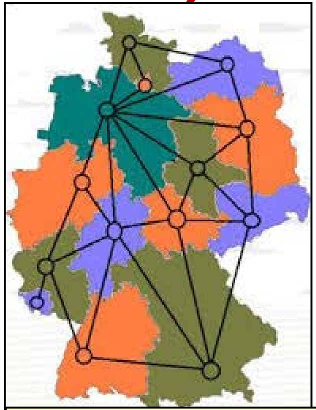
CRYPTARITHMETIC PUZZLE



N-QUEENs

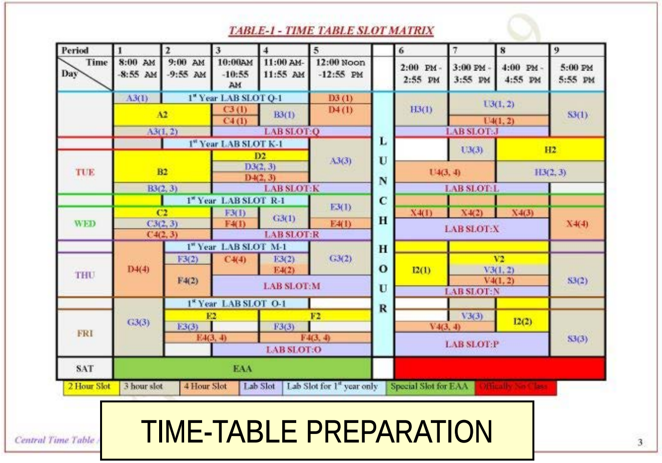


CROSSWORD PUZZLE

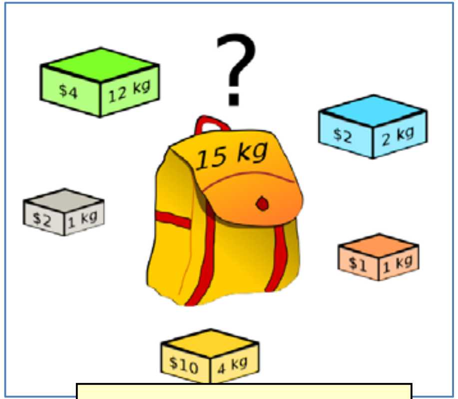


MAP COLOURING

AIRLINE GATE SCHEDULING



TIME-TABLE PREPARATION



KNAPSACK

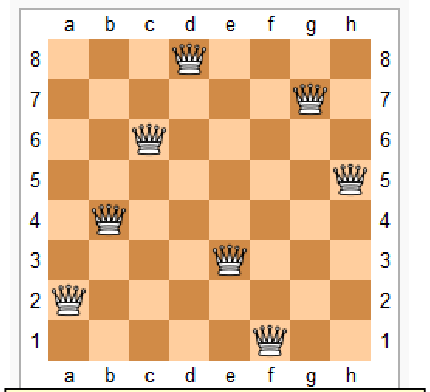
Basic CSP Formulation

- Variables
  - A Finite Set of Variables $V_1, V_2, \ldots, V_n$
- Domains
  - Each Variable has a Domain $D_1, D_2, \ldots, D_n$ from which it can take a value.
  - The Domains may be discrete or continuous domains
- Satisfaction Constraints
  - A Finite Set of Satisfaction Constraints, $C_1, C_2, \ldots C_m$
  - Constraints may be unary, binary or be among many variables of the domain
  - All Constraints have a Yes / No Answer for Satisfaction given values of variables
- Optimization Criteria (Optional)
  - A Set of Optimization Functions $O_1, O_2, \ldots O_p$
  - These Optimization Functions are typically max or min type
- Solution
  - To Find a Consistent Assignment of Domain Values to each Variable so that All Constraints are Satisfied and the Optimization Criteria (if any) are met.

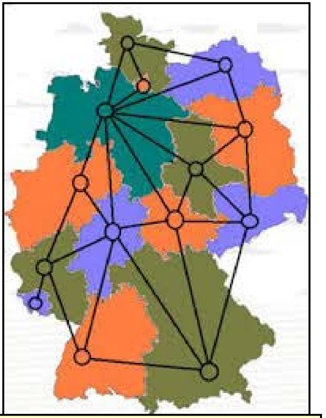
Formulating CSPs

```
      B O B
    x B O B
    -------
    M E O Y
  M I L O
M E O Y
-----------
M A R L E Y
```

CRYPTARITHMETIC PUZZLE



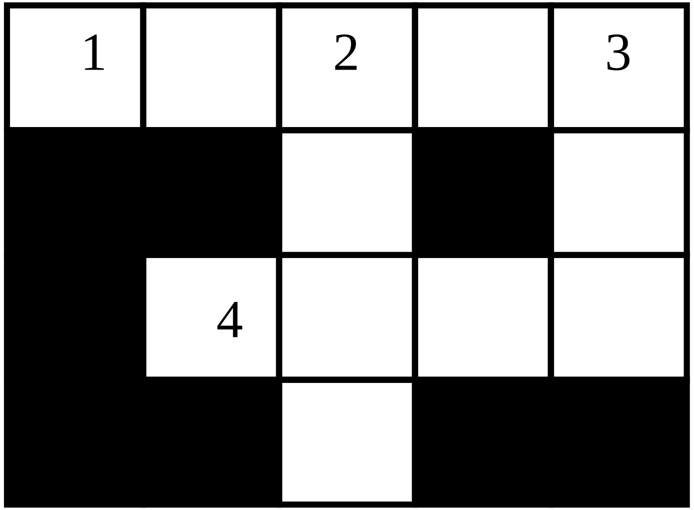
N-QUEENs



MAP COLOURING

1. VARIABLES
2. DOMAINS
3. SATISFACTION CONSTRAINTS
4. OPTIMIZATION CRITERIA
5. SOLUTION

Formulating CSPs: Crossword



<u>Word List:</u>

astar, happy, hello, hoses, live, load, loom, peal, peel, save, talk, ant, oak, old

1. VARIABLES
2. DOMAINS
3. SATISFACTION CONSTRAINTS
4. OPTIMIZATION CRITERIA
5. SOLUTION

Formulating CSPs: Flight Gate Scheduling

Flight No	Dep Time	G Start	G End
F1	7:00	6:15	7:15
F2	8:30	7:45	8:45
F3	7:45	7:00	8:00
F4	9:45	9:00	10:00
F5	10:00	9:15	10:15
F6	9:00	8:15	9:15
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1. VARIABLES
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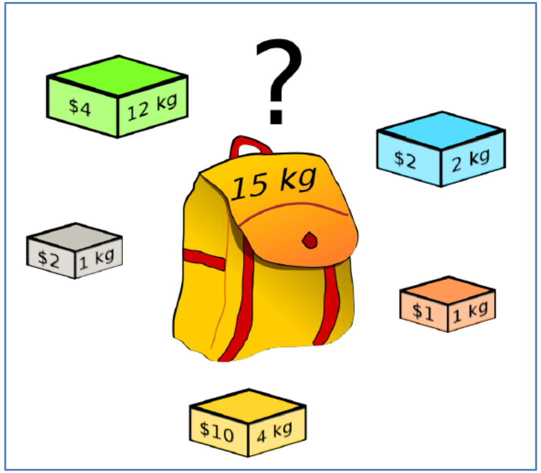
Formulating CSPs: Knapsack



1. VARIABLES
2. DOMAINS
3. SATISFACTION CONSTRAINTS
4. OPTIMIZATION CRITERIA
5. SOLUTION

Formulating CSPs: Time Table

1. VARIABLES
2. DOMAINS
3. SATISFACTION CONSTRAINTS
4. OPTIMIZATION CRITERIA
5. SOLUTION

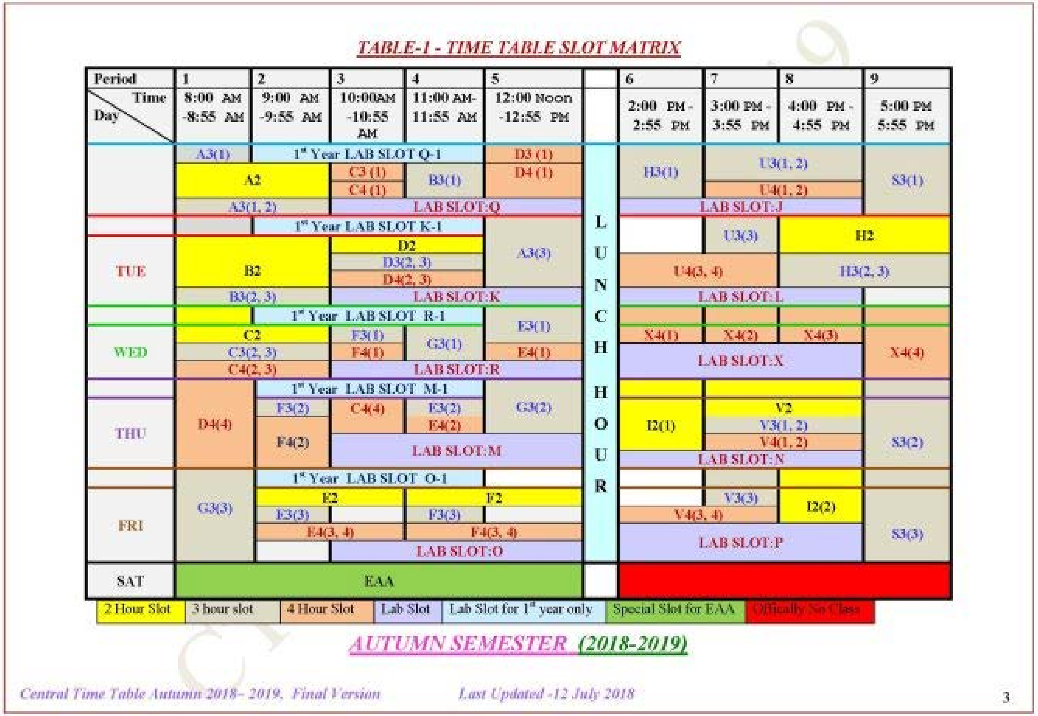


TABLE-1 - TIME TABLE SLOT MATRIX

Period / Time / Day	1 8:00 AM -8:55 AM	2 9:00 AM -9:55 AM	3 10:00AM -10:55 AM	4 11:00 AM-11:55 AM	5 12:00 Noon -12:55 PM		6 2:00 PM - 2:55 PM	7 3:00 PM - 3:55 PM	8 4:00 PM - 4:55 PM	9 5:00 PM 5:55 PM
	A3(1) A2 A3(1, 2)	1st Year LAB SLOT Q-1	C3 (1) C4 (1) LAB SLOT:Q	B3(1)	D3 (1) D4 (1)	L U N C H	H3(1) LAB SLOT:J	U3(1, 2) U4(1, 2)		S3(1)
TUE	B2 B3(2, 3)	1st Year LAB SLOT K-1	D2 D3(2, 3) D4(2, 3) LAB SLOT:K		A3(3)		U4(3, 4) LAB SLOT:L	U3(3)	H2 H3(2, 3)	
WED	C2 C3(2, 3) C4(2, 3)	1st Year LAB SLOT R-1	F3(1) F4(1) LAB SLOT:R	G3(1)	E3(1) E4(1)		X4(1) LAB SLOT:X	X4(2)	X4(3)	X4(4)
THU	D4(4)	1st Year LAB SLOT M-1 F3(2) F4(2)	C4(4) LAB SLOT:M	E3(2) E4(2)	G3(2)	H O U R	I2(1) LAB SLOT:N	V2 V3(1, 2) V4(1, 2)		S3(2)
FRI	G3(3)	1st Year LAB SLOT O-1 E2 E3(3) E4(3, 4)	LAB SLOT:O	F2 F3(3) F4(3, 4)			V4(3, 4) LAB SLOT:P	V3(3)	I2(2)	S3(3)
SAT	EAA									

2 Hour Slot	3 hour slot	4 Hour Slot	Lab Slot	Lab Slot for 1st year only	Special Slot for EAA	Officially No Class

AUTUMN SEMESTER (2018-2019)

Central Time Table Autumn 2018- 2019, Final Version — Last Updated -12 July 2018 — 3

Slots, Rooms, Subjects, Teachers, Students

Room-Slots: Subjects
Subjects: L-T-P, Teachers, Students

Multi-layered constraints

Intricate Optimization

Exercise: Time-Tabling in the era of online classes

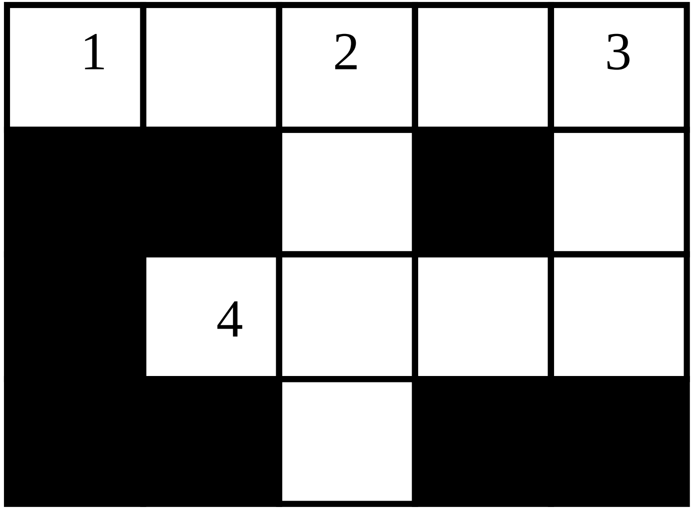
Formulating CSPs: Time Table

1. VARIABLES
2. DOMAINS
3. SATISFACTION CONSTRAINTS
4. OPTIMIZATION CRITERIA
5. SOLUTION

CSP Solution Overview

- CSP Graph Creation:
  - Create a Node for Every Variable. All possible Domain Values are initially Assigned to the Variable
  - Draw edges between Nodes if there is a Binary Constraint. Otherwise Draw a hyper-edge between nodes with constraints involving more than two variables
- Constraint Propagation:
  - Reduce the Valid Domains of Each Variable by Applying Node Consistency, Arc / Edge Consistency, K-Consistency, till no further reduction is possible. If a solution is found or the problem found to have no consistent solution, then terminate
- Search for Solution:
  - Apply Search Algorithms to Find Solutions
  - There are interesting properties of CSP graphs which lead of efficient algorithms in some cases: Trees, Perfect Graphs, Interval Graphs, etc
  - Issues for Search: Backtracking Scheme, Ordering of Children, Forward Checking (Look-Ahead) using Dynamic Constraint Propagation
  - Solving by Converting to Satisfiability (SAT) problems

CSP Graph for Crossword



Word List:
astar, happy, hello, hoses, live, load, loom, peal, peel, save, talk, ant, oak, old

CSP Graph for Airline Gate Scheduling

Flight No	Dep Time	G Start	G End
F1	7:00	6:15	7:15
F2	8:30	7:45	8:45
F3	7:45	7:00	8:00
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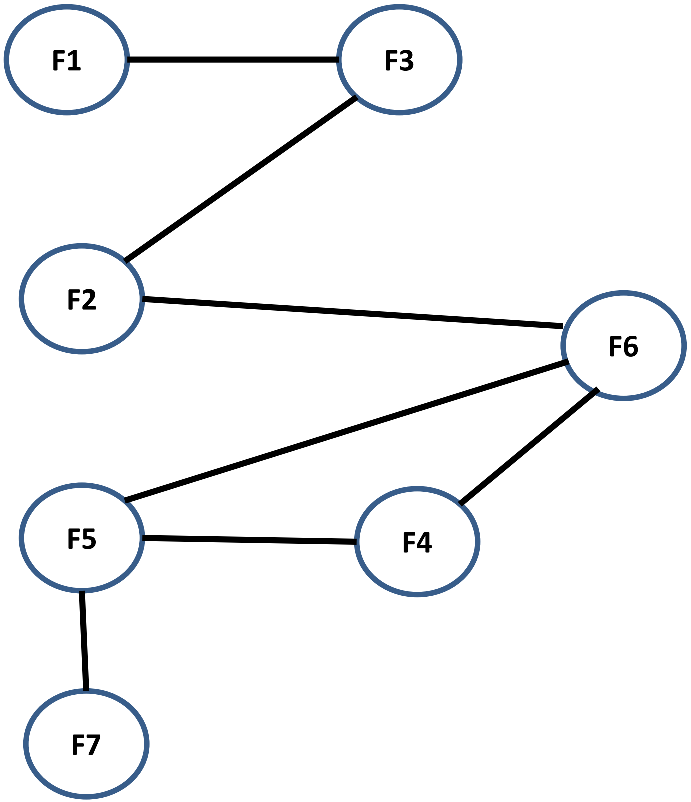
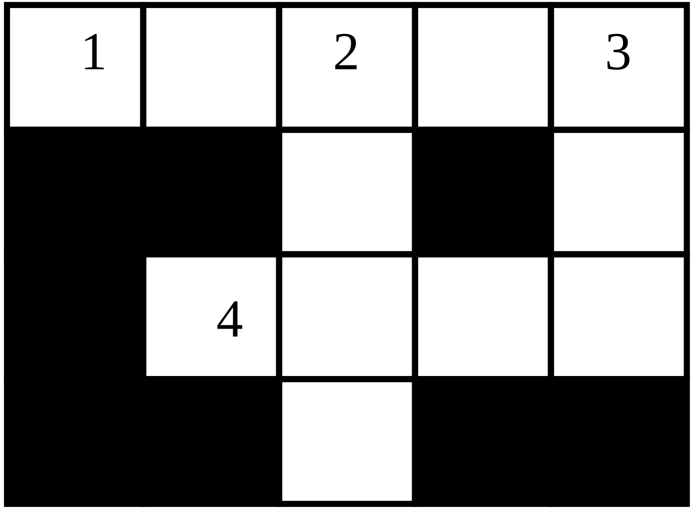


Constraint Propagation Steps

- Constraints
  - Unary Constraints or Node Constraints
  - Binary Constraints or Edges between CSP Nodes
  - Higher order or Hyper-Edges between CSP Nodes
- Node Consistency
  - For every Variable V_i, remove all elements of D_i that do not satisfy the Unary Constraints for the Variable
  - First Step is to reduce the domains using Node Consistency
- Arc Consistency
  - For every element x_ij of D_i, for every edge from V_i to V_j, remove x_ij if it has no consistent value(s) in other domains satisfying the Constraints
  - Continue to iterate using Arc Consistency till no further reduction happens.
- K-Consistency or Path Consistency
  - For every element y_ij of D_i, choose a Path of length L with L variables, use a consistency checking method similar to above to reduce domains if possible

CSP Graph for Crossword



Word List:

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Applying Node Consistency:

D1 = {astar, happy, hello, hoses}
D2 = {live, load, loom, peal, peel, save, talk}
D3 = {ant, oak, old}
D4 = {live, load, loom, peal, peel, save, talk}

NOW APPLY ARC CONSISTENCY

Applying Arc Consistency:

D1 = {astar, happy, hello, hoses}

D2 = {live, load, loom, peal, peel, save, talk}

D3 = {ant, oak, old}

D4 = {live, load, loom, peal, peel, save, talk}

Arc Consistency Algorithm AC-3

```
AC-3(csp) // inputs - CSP with variables, domains, constraints
  queue ← local variable initialized to all arcs in csp
  while queue is not empty do
      (X_i, X_j) ← pop(queue)
      if Revise(csp, X_i, X_j) then
          if size of D_i = 0 then return false
          for each X_k in X_i.neighbors-{X_j} do
              add (X_k, X_i) to queue
  return true
```

```
Revise(csp, X_i, X_j)
  revised ← false
  for each x in D_i do
      if no value y in D_j allows (x, y) to satisfy constraint between X_i and X_j then
          delete x from D_i
          revised ← true
  return revised
```

Time complexity: $O(n^2d^3)$

Consistency for Airline Gate Scheduling

Flight No	Dep Time	G Start	G End
F1	7:00	6:15	7:15
F2	8:30	7:45	8:45
F3	7:45	7:00	8:00
F4	9:45	9:00	10:00
F5	10:00	9:15	10:15
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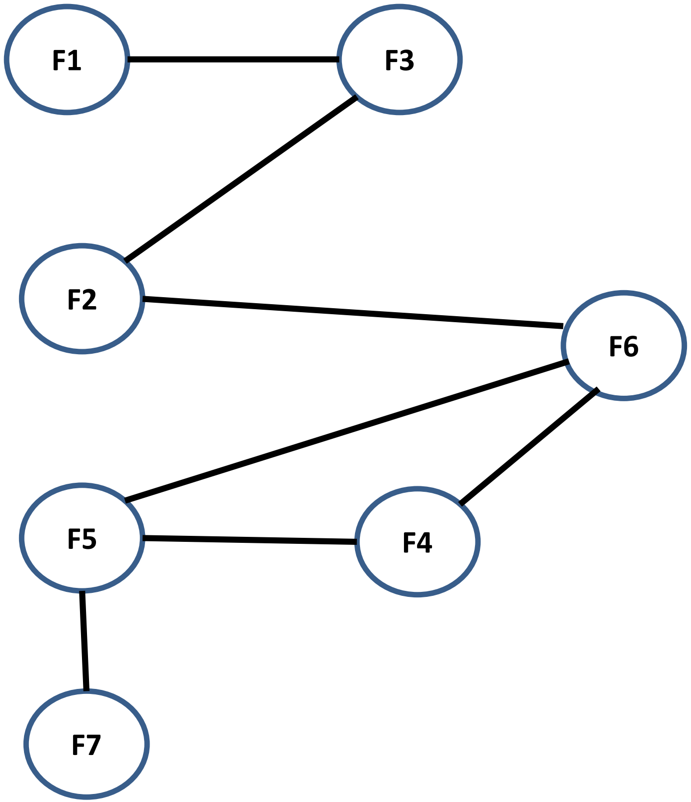

Backtracking Algorithm for CSP

CSP-BACKTRACKING({})

CSP-BACKTRACKING(a)

- If a is complete then return a
- X ← select unassigned variable
- D ← select an ordering for the domain of X
- For each value v in D do
  - If v is consistent with a then
    - Add (X= v) to a
    - result ← CSP-BACKTRACKING(a)
    - If result ≠ *failure* then return result
- Return *failure*

partial assignment of variables

Backtracking for Airline Gate Scheduling

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F1	7:00	6:15	7:15
F2	8:30	7:45	8:45
F3	7:45	7:00	8:00
F4	9:45	9:00	10:00
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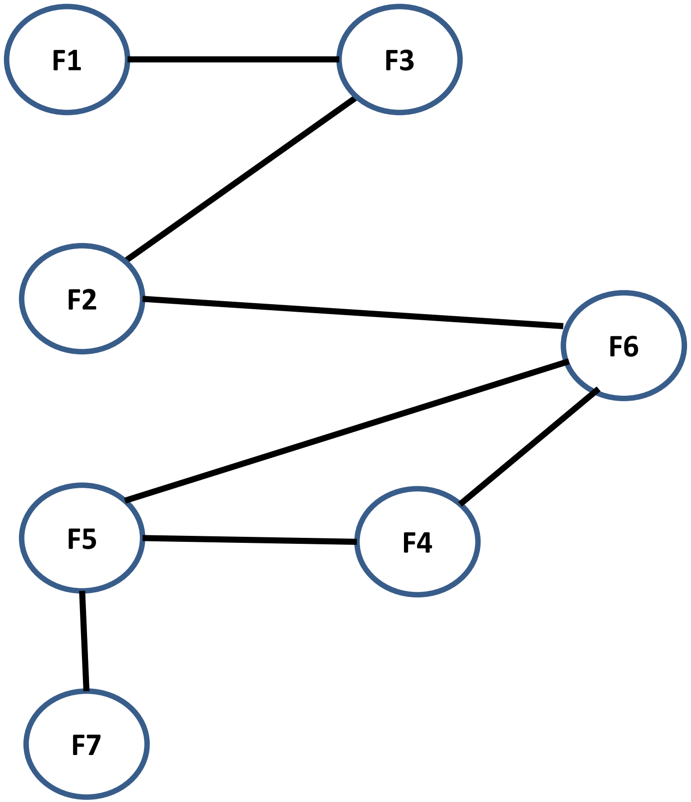
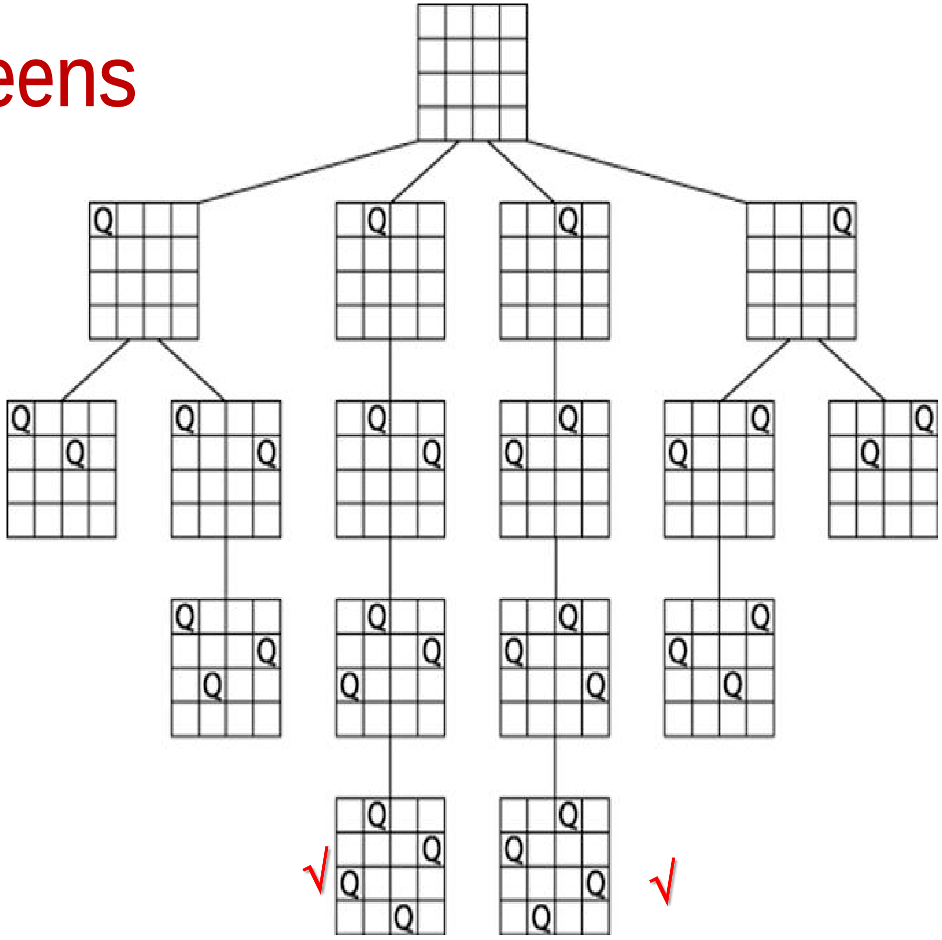


Search 4-Queens

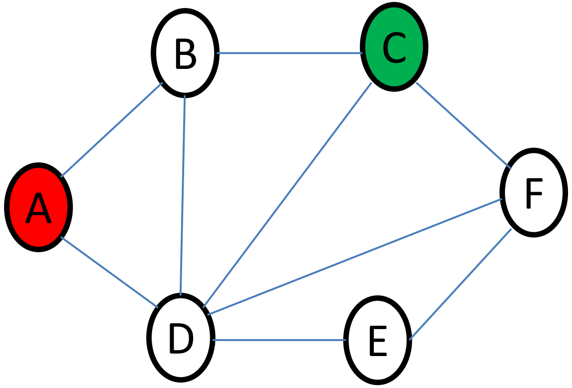


Strategies for CSP Search Algorithms

- Initial Constraint Propagation
- Backtracking Search
  - Variable Ordering
    - Most Constrained Variable / Minimum Remaining Values
    - Most Constraining Variable
  - Value Ordering
    - Least Constraining Value leaving maximum flexibility
  - Dynamic Constraint Propagation Through Forward Checking
    - Preventing useless Search ahead
  - Dependency Directed Backtracking
- SAT Formulations and Solvers
- Optimization
  - Branch-and-Bound
  - SMT Solvers, Constraint Programming
- Learning, Memoizing, etc
- CSP Problems are NP-Hard in General

Forward Checking: 3 Colouring Problem

- Forward checking propagates information from assigned to unassigned variables



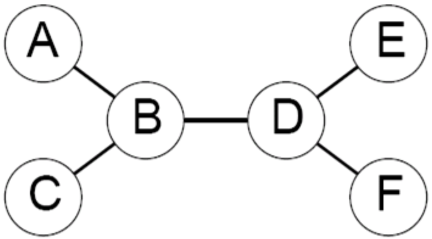
A	B	C	D	E	F
{R, G, B}	{R, G, B}	{R, G, B}	{R, G, B}	{R, G, B}	{R, G, B}
	{G, B}	{R, G, B}	{G, B}	{R, G, B}	{R, G, B}
	{B}		{B}	{R, G, B}	{R, B}

X

- B and D cannot both be blue!
- Why did we not detect this?
- Forward checking detects some inconsistencies, not all
- *Constraint propagation:* reason from constraint to constraint

Special Cases

Tree-structured CSPs



Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n\,d^2)$ time

Compare to general CSPs, where worst-case time is $O(d^n)$

For PERFECT GRAPHS, CHORDAL GRAPHS, INTERVAL GRAPHS, the Graph Colouring Problem can be solved in Polynomial Time

MITTTS Lecture Scheduling Problem

The MIT Time Travel Society (MITTTS) has invited seven famous historical figures to each give a lecture at the annual MITTTS convention, and you've been asked to create a schedule for them. Unfortunately, there are only four one-hour time slots available (1pm - 4pm), and you discover that there are some restrictions on how you can schedule the lectures and keep all the convention attendees happy.

For instance, physics students will be disappointed if you schedule Niels Bohr and Isaac Newton to speak during the same time slot, because those students were hoping to attend both of those lectures.

After talking to some students who are planning to attend this year's convention, you determine that they fall into certain groups, each of which wants to be able to see some subset of the timetraveling speakers. (Fortunately, each student identifies with at most one of the groups.) You write down everything you know:

The list of guest lecturers consists of Alan **T**uring, Ada **L**ovelace, Niels **B**ohr, Marie **C**urie,

Socrates, **P**ythagoas, and Isaac **N**ewton.

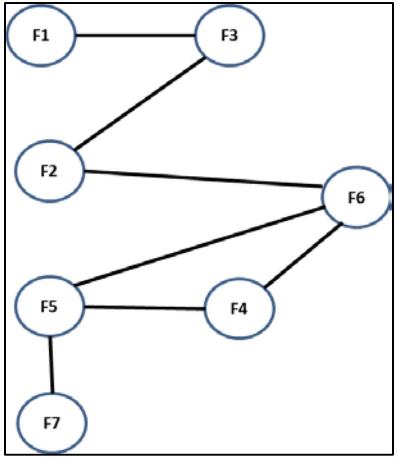
1. **T**uring has to get home early to help win World War II, so he can only be assigned to the 1pm slot.
2. The Course VIII students want to see the physicists: **B**ohr, **C**urie, and **N**ewton.
3. The Course XVIII students want to see the mathematicians: **L**ovelace, **P**ythagoras, and **N**ewton.
4. The members of the Ancient Greece Club wants to see the ancient Greeks: **S**ocrates and **P**ythagoras.
5. The visiting Wellesley students want to see the female speakers: **L**ovelace and **C**urie.
6. The CME students want to see the British speakers: **T**uring, **L**ovelace, and **N**ewton.
7. Finally, you decide that you will be happy if and only if you get to see both **C**urie and **P**ythagoras. (Yes, even if you belong to one or more of the groups above.)

You are to schedule these lectures satisfying the above constraints using minimum number of parallel sessions.

MITTTS Lecture Scheduling Problem

Solving CSP using SAT / SMT Solvers

- Boolean Satisfiability (SAT) is a CSP
- CSPs can be modelled as SAT problems
  - Try: Map Colour, Gate Scheduling, n-Queens
  - Home Exercise: Write a Generic Scheme to Convert and CSP Problem to a SAT Problem
- SAT has very efficient solvers
  - MiniSAT, CHAFF, GRASP, etc
- For Optimization cases, we can formulate them as
  - Satisfiability Modulo Theories (SMT) – with arithmetic and first order logic
  - 0/1 or Integer Linear Programming (ILP)
  - Constraint Programming Problems
  - SMT Solvers: Z3, Yices, Barcelogic, MathSAT, OpenSMT, etc



Thank you